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RESEARCH ARTICLE

Propagation of Love Waves in a Non-Homogeneous Elastic Media

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Abstract

The propagation of Love waves in a non-homogeneous layer of finite thickness lying over an isotropic semi-infinite medium is discussed in this study. It is assumed that density and rigidity are space dependent and obey the laws $\rho_1 = \rho_0\,(1+\cos\alpha z)$, $\mu=\mu_0\,(1+\cos\alpha z)$, where '$\alpha$' is a scaling parameter. The method of variable separation, substitution method for solving the second order partial differential equation has been used for finding the general solution of the problem. The dispersion of love waves is also discussed.

Keywords: Love waves, non-homogeneous layer, isotropic semi-infinite medium, differential equation.

Introduction

The earth has a layered structure and this exerts a significant influence on the propagation of elastic waves. The simplest cases of influence exerted on the propagation of seismic waves by a single plane boundary which separates two half-spaces with different properties, and by two parallel plane boundaries forming a layer. Earth is being treated as an elastic body in which three types of waves can occur.

1. Dilatational and equi-voluminal waves in the interior of the earth.
2. In the neighbourhood of its surface known as Rayleigh waves (1885).
3. Third type of waves occurs near the surface of contact of two layers of the earth known as love waves (1911).

The Rayleigh waves are observed far from the disturbance source near the surface. Since the energy carried by these waves is concentrated over the surface, its dissipation is slower than the Dilatational and equi-voluminal waves where the energy is dissipated over the volume of the disturbed region. Therefore, during earth quakes for an observer remote from the source of disturbance, the Rayleigh waves represent the greatest danger. In the case of Love waves, the energy is concentrated near the interface; hence they are dissipated more slowly. In the problem of propagation of Love type seismic wave in inhomogeneous isotropic media of finite depth lying over a infinite half space, it is shown that the distortional wave velocity in the layer is greater than in the semi infinite half space. The propagation of Love waves in an in homogeneous layer is of considerable importance in earth quakes engineering and seismology on account of occurrence of in homogeneities in the crust of the earth as the earth is supposed to be made up of different layers.

This problem has been studied by Sezawa (1935), Wilson (1942), Das and Gupta (1952), Deresiewicz (1961), Scholte (1961) by considering different models of a layer changing either density or rigidity and established the presence of Love Waves in each case. Tapan Kumar (1968) showed Love Waves in heterogeneous media. Chattopadhyay *et al.* (1986) employed Ghosh method (1963, 1970) to study Love waves excited by a source in a layer overlying inhomogeneous half-space. The problem of propagation of Love-type waves in layer deep in the earth, where the layer is either porous or inhomogeneous, has been studied by Harkride (1964), Hudson (1962), Lapwood (1948) and Kar (1977) among other. Paul (1964) studied the propagation of Love waves in a fluid saturated porous layer lying between two elastic half spaces. The analysis, however, in this case becomes a little complicated. Also Gogna (1976), Kausel (1986), Chadwick (1993), Zhang (1998) also studied the propagation of Love Waves through non-homogeneous media. Chiroiu and Ghiroiu (1992) investigated Love waves for a non-homogeneous elastic half-space.

In this study, we have studied the problem of propagation of seismic (Love type) waves in the earth which is composed of two physically different regions, one non-homogenous layer of variable rigidity and density, finite depth lying over homogeneous semi infinite half space. It has been shown that the phase velocity of the Love waves lies in between the wave velocities of the different regions such that the velocity of the upper layer is greater than the lower half space. It is also shown that dispersion takes place with respect to the wave length of the Love waves and accordingly dispersion takes place and also depends upon non-homogeneity scaling parameter.

Assumptions

1. It is assumed that for the parallel plane motion of a screen that all points belonging to the plane that are parallel to yz plane, the vibrations of the medium have the same phase and amplitude. They are determined by the coordinate x corresponding to plane ⊥ to x-axis i.e. we are considering time Harmonic propagation in the positive direction of x-axis.
2. It is assumed that the upper layer at the interface (screen) is welded contact with the lower half space. If we disturb the interface sufficiently rapidly in such a manner that it remains parallel to itself (plane parallel movement) then at my instant of time, the displacement at any of the interface (screen) will be same.
3. Cause of in homogeneity in the earth, as the geologically slow processes take place in the formation of loose sedimentary rocks tend to be heterogeneous and vary from place to place causing differences in their mechanical properties from point to point or layer to layer.
4. As the wave travels from the lower half space towards the upper layer their source of disturbance is at such as great distance that the waves may be considered as plane waves.

Formulation of problem

In linear theory of elasticity the stress-strain relations are given by

$$\sigma_{ij} = \lambda\theta\, \delta_{ij} + 2\mu\, \sigma_{ij} \quad \text{where } i, j = 1, 2, 3 \tag{1}$$

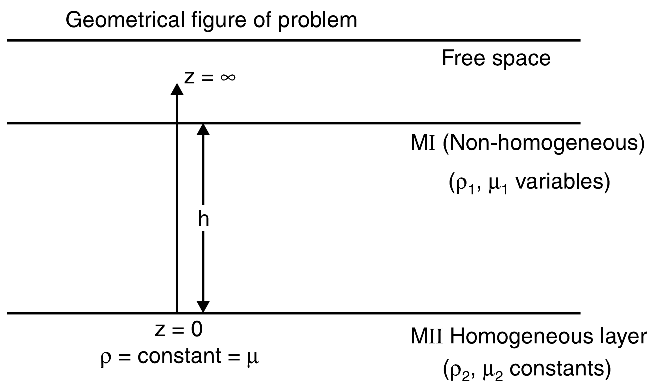
$$\text{Where} \quad e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{2}$$

$$\theta = \text{div } u_i \tag{3}$$

$$\delta_{ij} = \text{Kronecker delta function} = \begin{cases} 1 & ; i = j \\ 0 & ; i \neq j \end{cases} \tag{*}$$

$$(i, j = 1, 2, 3)$$

Fig. 1. Non-homogeneous layer on homogeneous layer.



Here λ, μ are called Lame's coefficients, i.e. constants for homogeneous medium and are space dependent for non-homogeneous medium.

Here the Domain of the problem is given as under

$$D(x, y, z) = \{(x, y, z); -\infty < x < \infty, -\infty < y < \infty, -\infty < z \leq h\}$$

$$\text{Where, } -\infty < z \leq h \quad (-\infty < z \leq 0) \cup (0 \leq z \leq h)$$

Here the problem of a heterogeneous isotropic layer of finite thickness h (M_I) lying over a homogeneous isotropic semi-infinite medium (M_{II}) is being considered. The two dimensional SH-Type wave motion in xz plane is considered with x-axis taken in the direction of propagation of disturbances and z-axis, is taken vertically upward and origin at the interface of the two media z=0. Now as the upper layer is heterogeneous in z-direction, so rigidity 'μ' and density 'ρ' are space-dependent and functions of coordinate 'z'.

$$\text{i.e.} \quad \mu_1 = \mu_0\, \mu_1(z) \text{ and } \rho_1 = \rho_0\, \rho_1(z) \tag{4}$$

(For non-homogenous upper layer M_I)

$$\mu_2 = \mu_1 = \mu_0 \text{ and } \rho_2 = \rho_1 = \rho_0$$

(at the interface z=0)

$$\text{and} \quad \mu_2 = \mu_0 \text{ and } \rho_2 = \rho_0 \tag{5}$$

(For homogeneous lower half space M_{II})

Where, μ_0 and ρ_0 are the constants of rigidity and density. Here z is the vertical distance from the interface z= 0 and h is the thickness of upper layer. Equations of motion in the absence of body forces are being expressed as

$$\sigma_{ij} = \rho\, u_{i,\,tt} \quad i, j = 1, 2, 3, \tag{6}$$

Where, $\sigma_{\iota\varphi}$ are stress components, ρ being the density of material which is constant for homogeneous medium (M_{II}) and space dependent for heterogeneous medium (M_I). In the present problem, for upper layer it is function of z and constant for lower half space. SH. Type disturbances are considered for which

$$u_i = (0, v, 0) \text{ where } v = v(x, o, z, t) \tag{7}$$

Equation of motion (6) and stress-strain relation (1) for SH-type disturbances and using (2), (3) and (*), are obtained as

$$\sigma_{xy,x} + \sigma_{yz,\,z} = \rho v,_{tt}\,, \tag{8}$$

$$\text{Where,} \quad \sigma_{xy} = \mu v,_{x}\,;\ \sigma_{yz} = \mu v,_{z}\,, \tag{9}$$

From (8) and (9); we get

$$(\mu v,_{x}),_{x} + (\mu v,_{z}),_{z} = \rho v,_{tt} \tag{10}$$

Now we use, $\varpi = v_1$ (x, o, z, t) for non-homogeneous upper layer (M_I) and v= v_2 (x, o, z, t) for homogeneous lower half space (M_{II}) (**)

Equation of motion (10) for upper non-homogeneous layer MI takes the form, by using eq (4)

$$\mu_1 v_{1,xx} + \mu_1 v_{1,zz} + v_{1,z} \cdot \frac{d\mu_1}{dz} = \rho_1 v_{1,tt},$$

$$\text{i.e. } \nabla^2 v_1 + \frac{1}{\mu_1} v_{1,z} \cdot \frac{d\mu_1}{dz} = \frac{\rho_1}{\mu_1} v_{1,tt}, \qquad (11)$$

Where, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$

Solution of problem

To solved eq. (11) we use

$$v_1 = V_1(z)\, e^{iK(x-ct)} = V_1(x, 0, z, t) \qquad (12)$$

For the forward wave propagation in the x-direction as harmonic functions of x and t. where K is the wave number and c is the unknown phase velocity of Love waves, which depends on wave length.

$$v_{1,xx} = V_1(z)\, e^{iK(x-ct)} (-K^2) \qquad (13)$$

$$v_{1,zz} = V_{1,zz}\, e^{iK(x-ct)} \qquad (14)$$

$$\text{and} \quad v_{1,tt} = V_1(z)\, e^{iK(x-ct)} (-K^2 c^2) \qquad (15)$$

With the help of (13)–(15), after cancelling the factor $e^{iK(x-ct)}$, the eq (11) reduces to

$$V_{1,zz} + \frac{1}{\mu_1} V_{1,z} \frac{d\mu_1}{dz} + K^2 \left(\frac{\rho_1}{\mu_1} c^2 - 1\right) V_1 = 0 \qquad (16)$$

To eliminate $V_{1,z}$ we use the transformation

$$V_1 = \left[\frac{1}{4\mu_1^2}(\mu_{1,z})^2 - \frac{1}{2\mu_1}(\mu_{1zz}) + K^2 c^2 \frac{\rho_1}{\mu_1} - K^2\right] V' \mu_1^{-1/2} \qquad (17)$$

Then, eq (16) reduces to

$$V'_{,zz} + V' = 0 \qquad (18)$$

The variation of rigidity and density for upper layer is taken as $0 \le z < h$

From eq (4) we have

$$\mu_1 = \mu_0, \quad \mu_1(z) = \mu_0 (1 + \cos \alpha z) \qquad (19)$$

$$\text{and } \rho_1 = \rho_0, \quad \rho_1(z) = \rho_0 (1 + \cos \alpha z) \qquad (20)$$

At interface $z = 0$, $\mu_1 = 2\mu_0$; $\rho_1 = 2\rho_0$

So that $\frac{\mu_1}{\rho_1} = \frac{\mu_0}{\rho_0}$ = constant α be the scaling parameter for density and rigidity using (19) and (20) in eq. (18); equation (18) reduces to

$$\Rightarrow \quad V'_{,zz} + \left[\frac{\alpha^2}{4} + K^2 \left(\frac{c^2}{m_1^2} - 1\right)\right] V' = 0 \qquad (21)$$

Where, $m_1^2 = \frac{\mu_1}{\rho_1} = \frac{\mu_0}{\rho_0}$ = constant, representing the characteristic velocity of transverse waves in the media (M_I) eq. (21) can be written as

$$V'_{,zz} + \xi^2 V' = 0, \qquad (22)$$

$$\text{Where,} \quad \xi^2 = \frac{\alpha^2}{4} + K^2 \left(\frac{c^2}{m_1^2} - 1\right) \qquad (23)$$

The solution of eq (22) is of the form

$$V' = (a_1 \cos\xi z + a_2 \sin\xi z)$$

Where, a_1, a_2 is arbitrary constants eq. (17) reduces to

$$V_1 = \frac{1}{\sqrt{2\mu_0}} (a_1 \cos\xi z + a_2 \sin\xi z) \sec\alpha \frac{z}{2} \qquad (24)$$

Now the eq. of motion for the lower homogeneous isotropic half space (media MII) is obtained as

$$\nabla^2 v_2 = \frac{1}{m_2^2} v_{2,tt} \qquad (25)$$

Where, $m_2^2 = \frac{\mu_2}{\rho_2}$ is the characteristic velocity of transverse waves in the homogenous half space and using eq. (5) and (**) To solve eq. (5) under the transformation

$$v_2 = V_2(z)\, e^{iK(x-ct)}, \qquad (26)$$

Eq. (25) reduces to

$$V_{2,zz} - K^2 \left(1 - \frac{c^2}{m_2^2}\right) V_2 = 0,$$

$$\text{or} \quad V_{2,zz} - N^2 V_2 = 0, \qquad (27)$$

$$\text{Where} \quad N^2 = K^2 \left(1 - \frac{c^2}{m_2^2}\right) \qquad (28)$$

Now eq. (27) is a differential equation of second order with constant coefficients so solution of eq. (27) is taken as

$$V_{2(z)} = A e^{Nz} + B e^{-Nz} \qquad (29)$$

The solution of (3.29) after satisfying the physical condition according to which harmonic type solution should be in dissipated form as $z \to \infty$ i.e. $V_2 \to 0$ as

$$V_{2(z)} = B e^{-Nz} \qquad (30)$$

Boundary conditions

1. At the interface z = 0 dividing the upper layer and the lower half space the waves may interact while displacement is independent of them. For inter-terrestrial conditions, it is assumed that the media are continues at interface, in other words the transmissions of disturbances through the interface should not slip relative to one another, at z = 0 given $V_1 = V_2$
2. It is also assumed that there is neither exfoliation nor formation of intermediary cavities at the interface during disturbances. As to stress, the interface z = 0 behaves, since the media are continuous like any other internal surface in a medium namely the stress on one side of it is same as that on the other side.
3. For the free surface, the interface z = 0 to a certain extent is being treated as a plane screens. This arbitrary plane parallel motion of the screen is the cause of generation of transverse waves on both sides of the screen.

The mathematical forms of boundary conditions are given as under:

i. $V_1 = V_2$ at interface $z = 0$ (31)

ii. $\mu_1 V_{1,z} = \mu_2 V_{2,z}$ at $z = 0$ (32)

iii. Vanishing of stress components at free surface z = h

iv. i.e. $\mu_1 V_{1,z} = 0$ at $z = h$ (33)

Putting the value of V_1 and V_2 from eq. (24) and eq. (30) in eq (31); we get

$$a_1 - \sqrt{2\mu_0}\,B = 0 \tag{34}$$

After Differentiating eq. (24) and eq. (30) w.r.t z, putting these values in eq. (32), we get

$$\xi a_2 + \sqrt{2\mu_0}\,NB = 0 \tag{35}$$

Again substituting the value of $V_{1,z}$ in eq. (33), we have

$$\frac{\alpha}{2}\left(a_1\cos\xi h + a_2\sin\xi h\right)\sec\frac{\alpha h}{2}\tan\frac{\alpha h}{2} + \sec\frac{\alpha h}{2}\,[-a_1\xi\sin\xi h$$

$$+ a_2\,\xi\cos\xi h] = 0, \quad \text{as } \sec\frac{\alpha h}{2} \neq 0$$

$$\therefore\ a_1(H - \xi\tan\xi h) + a_2(H\tan\xi h + \xi) = 0 \tag{36}$$

Where, $H = \frac{\alpha}{2}\tan\frac{\alpha h}{2}$

Eliminating a_1, a_2 and B from equations (34), (35) and (36), we get

$$\begin{vmatrix} H - \xi\tan\xi h & H\tan\xi h + \xi & 0 \\ 1 & 0 & -\sqrt{2\mu_0} \\ 0 & \xi & \sqrt{2\mu_0}\,N \end{vmatrix} = 0$$

$$(H - \xi\tan\xi h)(\xi) - (H\tan\xi h + \xi)(N) = 0$$

$$\therefore\quad \tan\xi h = \frac{H\xi - N\xi}{NH + \xi^2}$$

Since arc tangent is a multiple valued function

$$\therefore\ \xi h = n\pi + \tan^{-1}\theta$$

Where $\theta = \dfrac{H\xi - N\xi}{NH + \xi^2}$

$$\theta = \frac{\left[\frac{\alpha}{2}\tan\frac{\alpha h}{2} - K\sqrt{1 - \frac{c^2}{m_2^2}}\right]\sqrt{\frac{\alpha^2}{4} + K^2\left(\frac{c^2}{m_1^2} - 1\right)}}{\frac{\alpha}{2}\tan\frac{\alpha h}{2} + K\sqrt{1 - \frac{c^2}{m_2^2} + \frac{\alpha^2}{4} + K^2\left(\frac{c^2}{m_1^2} - 1\right)}}$$

Case I

For homogeneous media $\alpha = 0 \Rightarrow H = 0$

$$\theta = -\frac{N}{\xi} = \frac{-K\sqrt{1 - \frac{c^2}{m_2^2}}}{K\sqrt{\frac{c^2}{m_1^2} - 1}}$$

i.e. $$\theta = \frac{-\frac{c\lambda_{m_1}}{m_2}\sqrt{\frac{m_2^2}{c^2} - 1}}{\frac{c\lambda_{m_2}}{m_1}\sqrt{1 - \frac{m_1^2}{c^2}}} = -\frac{m_1\lambda_{m_1}\sqrt{\frac{m_2^2}{c^2} - 1}}{m_2\lambda_{m_2}\sqrt{1 - \frac{m_1^2}{c^2}}}$$

Where, $m_1 = \sqrt{\frac{\mu_1}{\rho_1}}$; $m_2 = \sqrt{\frac{\mu_2}{\rho_2}}$

$$\therefore\ \frac{2\pi h}{\lambda m_1}\sqrt{\frac{c^2}{m_1^2} - 1} = n\pi + \arctan\left[\frac{m_1\lambda_{m_1}}{m_2\lambda_{m_2}}\sqrt{\frac{\frac{m_2^2}{c^2} - 1}{1 - \frac{m_1^2}{c^2}}}\right] \tag{37}$$

Here, n is an integer determining the mode number; for n = 0, we get the fundamental mode.

If we put $c = m_2$ in (37), then the quantity $\frac{\lambda_{m_1}}{h}$ in contrast to the fundamental mode where it is infinite – has a maximum limiting value equal to $\frac{2}{n}\sqrt{1-\frac{m_1^2}{m_2^2}}$. Thus for the mode number n, the variation of $\frac{\lambda_{m_1}}{h}$ corresponding to all possible values $m_1 \le c \le m_2$ only occurs in the limits $0 \le \frac{\lambda_{m_1}}{h} \le \frac{2}{n}\sqrt{1-\frac{m_1^2}{m_2^2}}$ and, as the mode number increases, this interval of existence of the dispersion curve decreases. The same dispersion curve for the fundamental mode compresses in the horizontal direction, which can be seen in Fig. 2.

When n = 1 and n = 2, the shape of the dispersion curve for normal mode reduces to

$$\frac{Y^2}{\frac{1}{4}} - \frac{X^2}{\frac{1}{n^2}} = 1, \text{ which shows a hyperbola.}$$

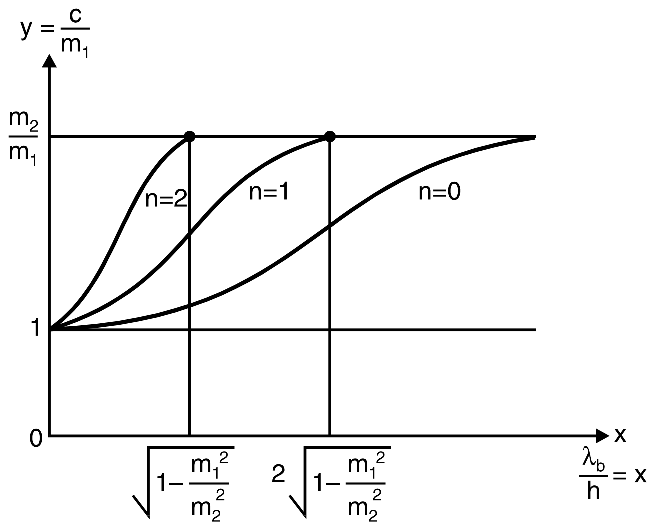
Where $Y = y^2 - \frac{1}{2} = \left(\frac{c}{m_1}\right)^2 - \frac{1}{2}$; $X = \frac{\lambda_{m_1}}{h}$

For $n = 0$; $c = m_1$

For $n = \infty$, wave does not exists.

We have shown that the ray scheme can be applied to propagation of plane waves in a layer. In particular, applying this scheme we obtained a dispersion equation for the layer lying on an absolutely rigid half-space.

Fig. 2. Propagation of plane waves in a layer.



Case-II

For non-homogeneous $\alpha \neq 0$, $H \neq 0$, we have

$$h\left\{\frac{\alpha^2}{4}+\frac{4\pi^2}{\lambda_{m_1}^2}\left(\frac{c^2}{m_1^2}-1\right)\right\}^{1/2} = \xi h$$

$$= n\pi + \arc\tan \frac{\left\{\frac{\alpha}{2}\tan\frac{\alpha h}{2} - \frac{2\pi}{\lambda_{m_2}}\sqrt{1-\frac{c^2}{m_2^2}}\right\} \times \left\{\frac{\alpha^2}{4}+\frac{4\pi^2}{\lambda_{m_1}^2}\left(\frac{c^2}{m_1^2}-1\right)\right\}^{1/2}}{\frac{\alpha}{2}\tan\frac{\alpha h}{2} \times \frac{2\pi}{\lambda_{m_2}}\sqrt{1-\frac{c^2}{m_2^2}} + \frac{\alpha^2}{4} + \frac{4\pi^2}{\lambda_{m_1}^2}\left(\frac{c^2}{m_1^2}-1\right)}$$

For argument of arc tangent, we have

$$\frac{\alpha}{2}\tan\frac{\alpha h}{2} - \frac{2\pi}{\lambda_{m_2}}\sqrt{1-\frac{c^2}{m_2^2}} = 0$$

$$\text{i.e.} \quad 1-\frac{\lambda_{m_2}^2 H^2}{4\pi^2} = \frac{c^2}{m_2^2} \Rightarrow c = m_2\sqrt{1-\frac{\lambda_{m_2}^2 H^2}{4\pi^2}}$$

Therefore, the shapes of the dispersive curves for the second media are ellipses.

Conclusion

1. Phase velocity 'c' depends upon α, the scaling parameter of non-homogeneous layer (M_I)
2. Phase velocity 'c' depends upon m_2 i.e. the characteristic velocity of waves in homogeneous half space M_{II}
3. The relation between the ratio of velocities and wave-length is elliptic.

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